

Experiments on the confinement of an electron gas in a magnetic field

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With 9 figures in the text

ABSTRACT

The confinement of electrons in different types of magnetic "bottles" is studied experimentally. A cathode is placed inside the confinement region and the electron current to the surrounding walls is measured as a function of the magnetic field strength. Toroidal and mirror-shaped fields are examined as well as the field generated by a circular current loop.

The experiments indicate that the field generated by the current loop can be used to trap particles in all directions. Further, it is found that the suppression of the total electron current is controlled only by the poloidal part of the magnetic field, as predicted by theory.

I. Introduction

Confinement of charged particles by a magnetic field is one of the central problems in plasma physics and especially in research on controlled fusion. However, as a plasma is built up of single charged particles, it is fundamental to inquire if such particles can be confined. This forms a first step in a chain of investigations which also concerns interactions between particles and stability problems.

Since the ions are much heavier than the electrons, weaker magnetic fields can be used for confinement in electron experiments than in plasma experiments. In the present investigation electrons have been evaporated from a hot cathode into different magnetic field configurations. The current between the cathode and the various parts of the apparatus has been measured as a function of the magnetic field strength. Measurements of this type have been performed earlier in the case of the magnetron as described by Harvey (1943). In the magnetron the current is controlled mainly by Hull's cut-off relation. The aim of the present investigations is mainly to study the particle confinement in the magnetic field from a ring current, as suggested by Lehnert (1958, 1959).

II. The cut-off relation

Consider an electromagnetic field distribution which is symmetric around the z -axis of a cylindrical coordinate system (r, φ, z) . The electrostatic field can be described by the potential $V(r, z)$ and the magnetic field by the vector potential $\mathbf{A}(r, z)$, so

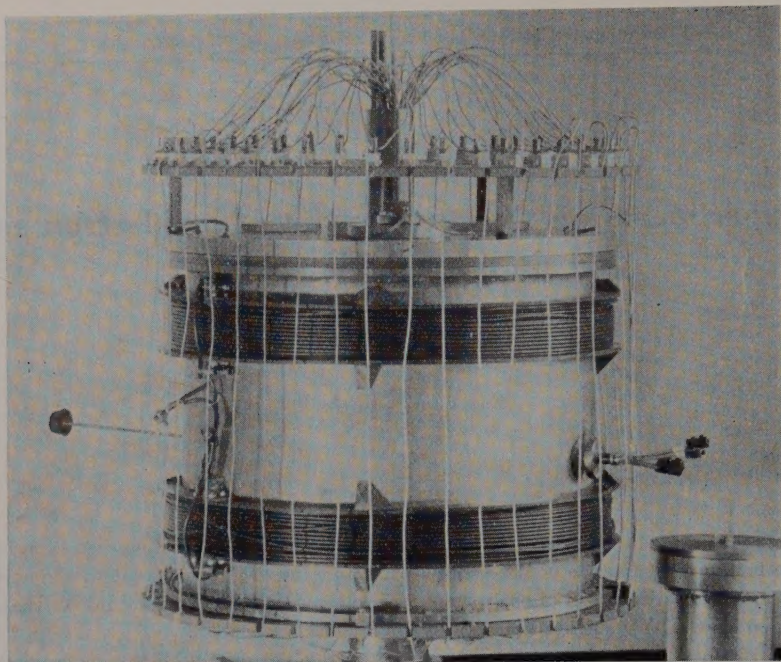


Fig. 1. The apparatus. The figure shows the winding producing the toroidal field and the Helmholtz coil pair.

that V and A are independent of φ . Then, it is possible to show (e.g. Glaser 1952, Lehnert 1959) from the constancy of total energy and of generalized momentum that

$$v_r^2 + v_z^2 = v_0^2 + \frac{2q}{m} (V_0 - V) - \frac{1}{r^2} \left[r_0 v_{\varphi_0} + \frac{q}{m} (r_0 A_{\varphi_0} - r A_{\varphi}) \right]^2, \quad (1)$$

where v is the particle velocity and the subscript $(_0)$ refers to the starting point of the particle, whose charge and mass are denoted by q and m . As $v_r^2 + v_z^2 \geq 0$, this relation can be used to compute the forbidden regions of the particle. It reduces to Hull's cut-off expression in the case of a homogeneous magnetic field. It should be observed that in this relation the magnetic field is represented only by A_{φ} , i.e., the relation contains only the poloidal magnetic field component and not the toroidal component. The cut-off depends only on the electrode potentials and is independent of space-charge effects, if the charge distribution is axially symmetric. It should also be observed that the motion of a particle in the r - z plane may be considered to take place in a "trough" given by the potential

$$U = V - V_0 + \frac{m}{2qr^2} \left[r_0 v_{\varphi_0} + \frac{q}{m} (r_0 A_{\varphi_0} - r A_{\varphi}) \right]^2, \quad (2)$$

which is independent of the toroidal magnetic field component.

III. Apparatus

Figs. 1, 2 and 5 show the apparatus within which different magnetic field configurations can be generated. A pair of coils, as shown in Fig. 2, has been used to create a mirror field; another pair is arranged as a Helmholtz coil pair and gives an approximately homogeneous field. The circular coil inside the chamber gives a poloidal field. Finally, current along the axis of the apparatus gives rise to a toroidal field.

The cathode system has been designed so that the measured cut-off curves are not related to the geometry of the cathode, i.e., its dimensions have been kept as small as possible.

Only low voltages were applied in order to avoid ionisation processes in the residual gas which would otherwise give rise to additional currents. The cathode currents were of the order of $1\ \mu\text{A}$ and they were measured with mirror galvanometers.

A rough estimate of the densities of the electrons and the neutral particles shows that formation of negative ions need not be taken into account in the present experiments.

Further, the influence of the earth's magnetic field (≈ 0.5 gauss) was examined in a special experiment. A large Helmholtz coil pair could be used to neutralize the earth's magnetic field over the whole apparatus. When this was used and no other

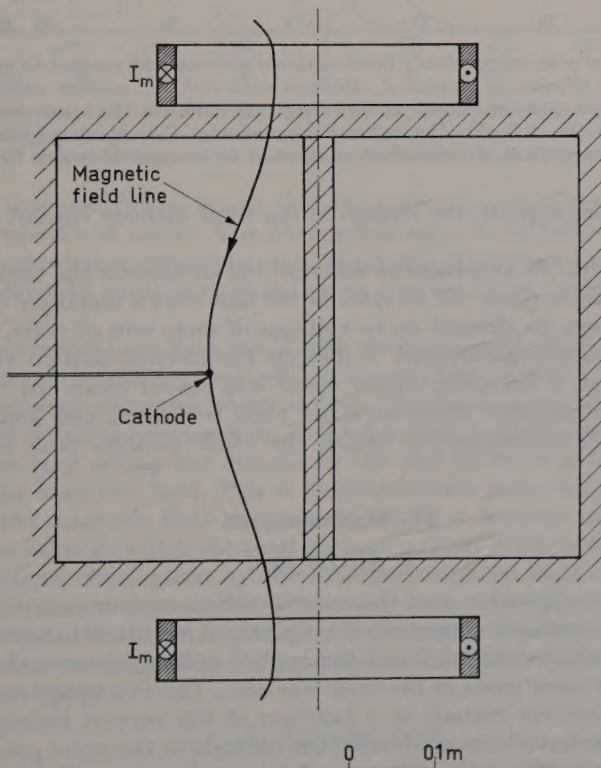


Fig. 2. Section through the apparatus as used in the measurements with the mirror field.

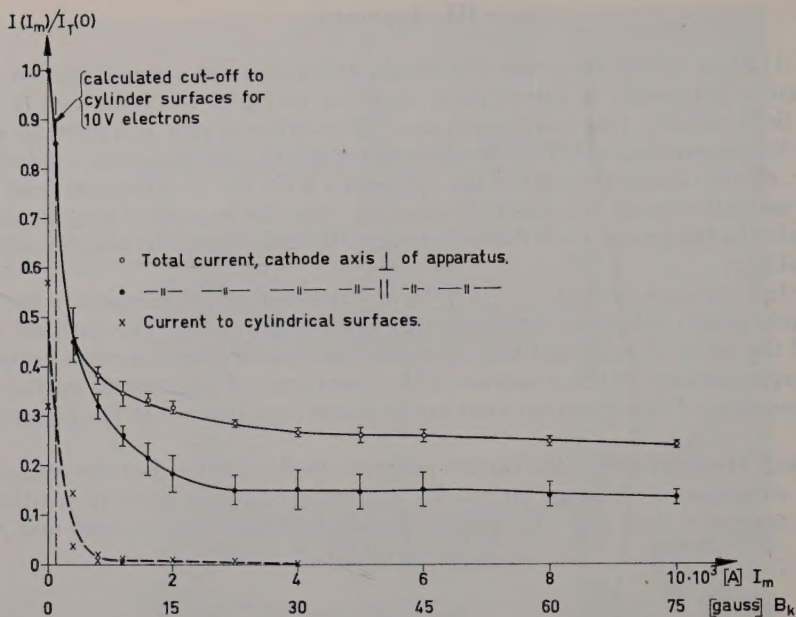


Fig. 3. Measurements with mirror field. Total cathode current and current to cylindrical surfaces as functions of total circulating current I_m through each coil. All current values on y -axis are shown relative to total cathode current at zero magnetic field. On the lower scale is shown B_k , the magnetic field at the cathode. The vertical line indicates the calculated cut-off value with respect to the cylinder surfaces. Total cathode current at zero magnetic field is $20 \mu A$.

magnetic fields were applied, the change in the total cathode current was less than 5 %.

One difficulty with the experiments was that the surfaces of the tank became contaminated by a film of pump oil, in spite of the fact that a liquid air trap was used. Films of this sort can be charged up to voltages of some tens of volts. If this charge becomes asymmetrically distributed, it distorts the electron motion very much. To eliminate this effect, a bakeable copper vessel was placed inside the vacuum tank. All surfaces which collected electrons could then be heated and kept fairly clean without the trouble encountered in baking the whole vacuum tank with its O-ring seals.

IV. Measurements

1. Mirror field

A section of the apparatus and the corresponding measurements are shown in Figs. 2 and 3. The cathode consisted of a directly d.c.-heated tungsten spiral. The voltage over the cathode was 4.5 V and the applied voltage between the positive end of the cathode and other parts of the tank was zero. The two upper curves show the total normalized cathode current as a function of the current through the mirror coils. Measurements have been made with the cathode in the same position but with different orientations. Thus, the difference between the curves is due to anisotropy of the cathode. The lower dashed curve shows the current to the cylindrical surfaces.

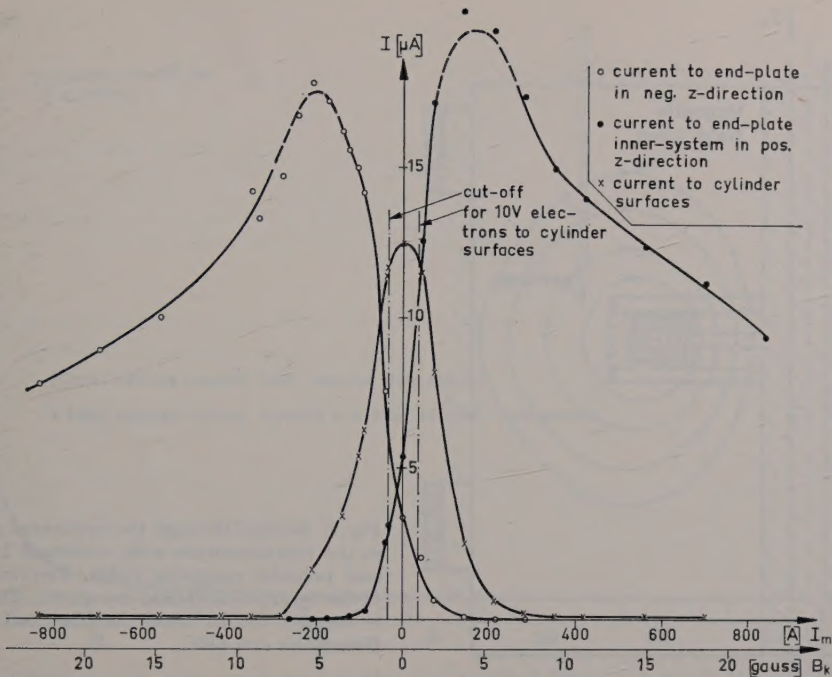


Fig. 4. Measurements with toroidal field. Current to different parts of the apparatus as a function of total current I_m through the central conductor. Cathode placed 7.6 cm from the axis, 24.7 cm from the top and 17.7 cm from the bottom of the apparatus.

For a strong magnetic field it is seen that this current is negligible and that all current went to the plane end plates. The phenomena seem to be fully developed at about 2000 A total circulating current in each mirror coil. This gives a magnetic field at the cathode of about 15 gauss, corresponding to a radius of curvature for 10 V electrons of 0.7 cm.

The measurements have been made with strong magnetic fields compared with the calculated magnetic field for "cut-off" to the cylinder surfaces. To get the expected result of a vanishing current to these surfaces, one has to apply much stronger magnetic fields than simple theory predicts. It is interesting to note that when the magnetic field becomes very strong the current to the end plates is independent of further changes in the magnetic field. This is understandable since the velocity of the electrons along the magnetic field lines should not be a function of the magnetic field strength. The value to which the total current is diminished depends on two factors. One is the mirror effect given by the stronger magnetic field at the end plates as compared with that at the cathode. The second is the reduced solid angle of the anode as seen from the cathode; at zero magnetic field both the cylinder surfaces and the end plates act as anodes, but at strong magnetic fields there is a measurable current only to the end plates.

2. Toroidal field

The measurements with a toroidal field are shown in Fig. 4. The apparatus was assembled in principle as in Fig. 5. It was made up of a cathode of the same type as

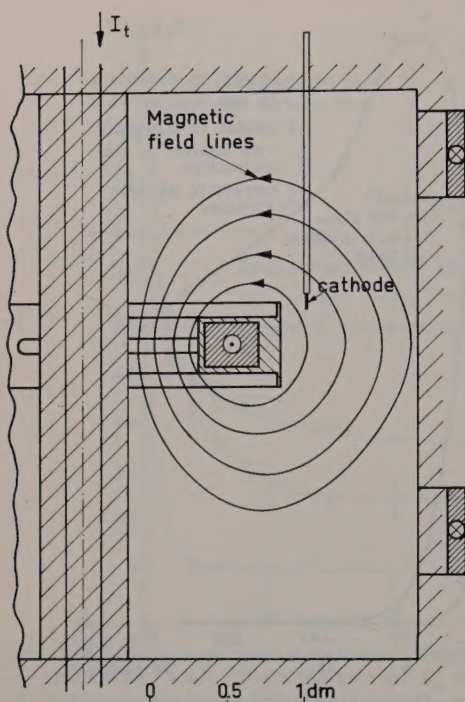


Fig. 5. Section through the apparatus as used in the measurements with combined toroidal and poloidal magnetic fields. The field lines of the total poloidal field are shown. This field is created by an inner circular coil and a Helmholtz coil pair.

described in section 1 together with three separate anodes. The latter consisted of (1) the inner and outer cylinder surfaces connected together, (2) the bottom plate, and (3) the top plate together with the ring structure.

The toroidal magnetic field was produced by a current through the vertical central rod. Current to the anodes was measured as a function of this current. Qualitatively the curves behave as expected, i.e., only with weak toroidal magnetic field is there a current flow to the cylinder surfaces. In stronger fields all electrons drift in a direction opposite to the current along the central rod. As the drift velocity in an inhomogeneous magnetic field is reduced when the magnetic field increases, the curves fall off at higher field strengths. The asymmetries in the curves are probably due to the geometrical asymmetry in the apparatus described. The magnetic field at the cathode for a total toroidal current of 200 A equals 5.3 gauss giving a radius of curvature of 2 cm for 10 V electrons and this corresponds approximately to the cut-off line shown in Fig. 4.

Apparently, we have here a case where a magnetic field gradient gives rise to a current. In macroscopic plasma theory it is shown that such a gradient will not generate a macroscopic current when the density of particles is constant. Therefore, if this point of view is adopted, one can say instead that the magnetic field gradient in this case gives rise to a density gradient. This will, in its turn, produce an electric current.

3. Field from a current loop

The apparatus is shown in Fig. 5. The inner circular coil producing the main poloidal field was suspended by three horizontal rods fixed to a vertical central steel pipe.

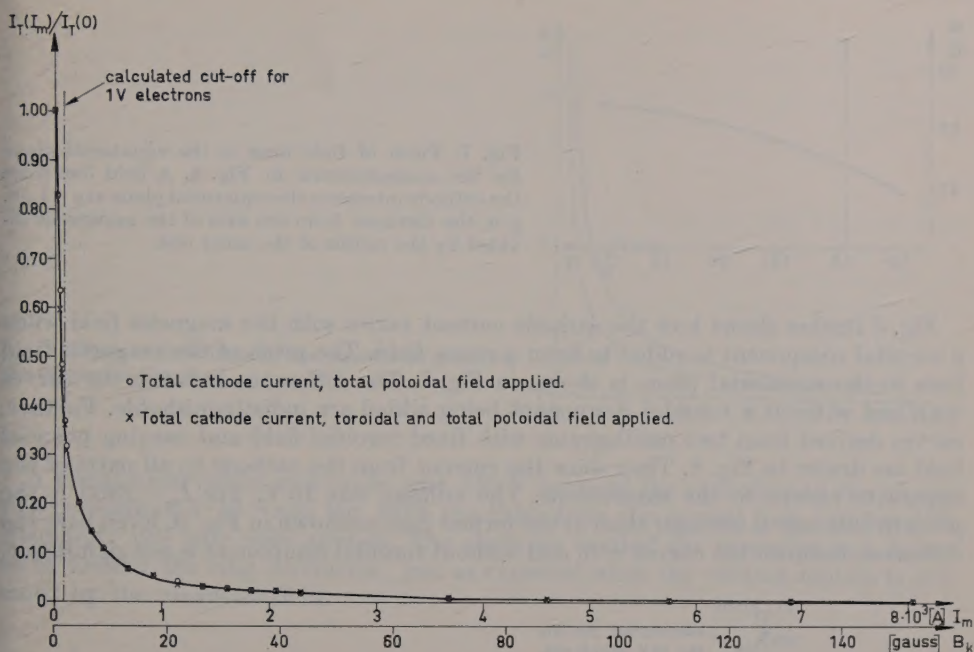


Fig. 6. Measurements with combined poloidal and toroidal fields. Total cathode current as a function of the total circulating current in the inner circular coil. Total circulating current in the Helmholtz coil pair = $0.26 I_m$. Toroidal current = $0.183 I_m$. Total cathode current at zero magnetic field is $4 \mu\text{A}$.

Also shown in the figure are the Helmholtz coil pair. The field from these is arranged in such a way that it strengthens the main field in the region between the inner coil and the outer cylindrical surface. It was also possible to apply a toroidal field by a winding passing through the central steel pipe.

Fig. 6 shows measurements of the normalized total cathode current as a function of I_m . The cathode was indirectly heated and placed as in Fig. 5. The magnetic field strength at the cathode was 1.9 gauss at $I_m = 100$ A. Cut-off for 1 eV electrons is calculated from eq. (1) and shown in the figure. With the total poloidal field from the inner and the Helmholtz coil pair applied, the cathode current falls off very fast with increasing magnetic field. However, just as described in the case of the mirror field, the current to the cylinder surfaces does not vanish at the calculated cut-off. For the curves just described the applied voltage was zero and the current to the suspensions of the ring was not measurable. This is understandable since the ratio between the magnetic fields at the suspensions and at the cathode was 5.9, corresponding to a strong mirror effect for electrons moving from the cathode to the suspensions. In addition, the geometrical cross-section of the suspensions is some 10^3 times smaller than the vessel surface, and receives a very small current at zero magnetic field. If +10 V was applied to the vessel and the suspensions with respect to the cathode, current could be measured to the suspensions only in the presence of a magnetic field. When the magnetic field is switched on, the magnetic mirror effect is compensated by the electric field which accelerates particles along the magnetic field lines.

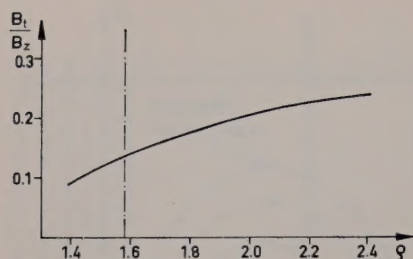


Fig. 7. Pitch of field lines in the equatorial plane for the measurements in Fig. 6. A field line from the cathode intersects the equatorial plane at $q = 1.58$. q is the distance from the axis of the apparatus divided by the radius of the inner coil.

Fig. 6 further shows how the cathode current varies with the magnetic field when a toroidal component is added to form a screw field. The pitch of the magnetic field lines in the equatorial plane is shown in Fig. 7. The difference between the curves with and without a toroidal component being added are indistinguishable. Further, curves derived from two oscillograms with fixed toroidal field and varying poloidal field are drawn in Fig. 8. They show the current from the cathode to all parts of the apparatus except to the suspensions. The voltage was 10 V. For $I_m < 2300$ A the pitch in this case is stronger than in the former case as shown in Fig. 9. Even here the difference between the curves with and without toroidal component is not significant.

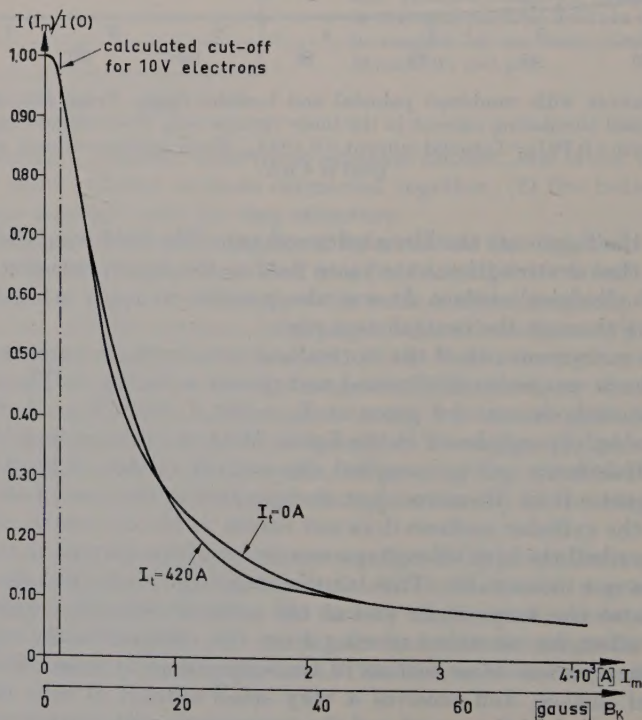


Fig. 8. Curves drawn from oscillograms showing total cathode current as a function of the total circulating current in the inner circular coil. Total circulating current in the Helmholtz coil pair = $0.26 I_m$. Total toroidal current 0 A and 420 A. Radial distance between cathode and axis 15.6 cm, axial distance from equatorial plane 4.6 cm.

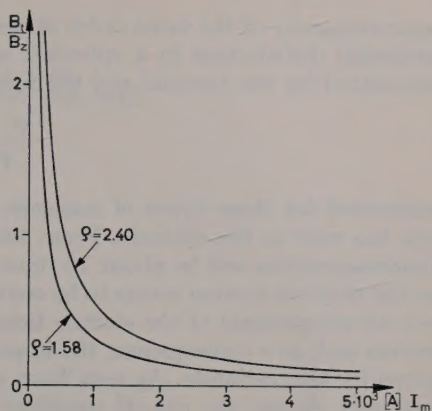


Fig. 9. Pitch of field lines in the equatorial plane for the curves in Fig. 8.

A visual observation was made with 150 V between the cathode and the anodes at a pressure of $3 \cdot 10^{-4}$ mm Hg. Both the fields from the current loop and from the Helmholtz coil pair were applied. The electrons excited gas molecules in a hollow torus around the ring conductor, just as expected when the electron motion is governed by the magnetic field.

V. Discussion of the method of measurement

To get a measurable cathode current a small voltage has in some cases been applied between the cathode and the rest of the apparatus. The cathode was small compared with other dimensions of the apparatus. The electric field was therefore concentrated to the cathode region, and nearly the whole potential drop occurs there. Consequently, the electrons are accelerated there and when they have left that region they will move with nearly constant energy. The drift motion in crossed electric and magnetic fields might modify the particle motion as compared with the motion of a mono-energetic particle in a magnetic field. The drift velocity in crossed electric and magnetic fields is

$$\mathbf{u}_E = \frac{1}{B^2} [\mathbf{B} \times \nabla V];$$

the drift velocity in an inhomogeneous magnetic field is

$$\mathbf{u}_{\perp B} = \frac{m w_{\perp}^2}{2 e B^3} \mathbf{B} \times \nabla B.$$

The ratio between the drift velocities then becomes

$$\frac{u_{\perp E}}{u_{\perp B}} = \frac{e V}{\frac{1}{2} m w_{\perp}^2} \cdot \left| \frac{\nabla_{\perp} V}{V} / \frac{\nabla_{\perp} B}{B} \right|.$$

Here V is the voltage between the cathode and the other parts of the apparatus, $w_{\perp}$ is the particle velocity perpendicular to the magnetic field. eV and $\frac{1}{2} m w_{\perp}^2$ are

approximately of the same order of magnitude. $\nabla_{\perp} V/V$ has been estimated from the potential distribution in a spherical space charge limited diode. $\nabla_{\perp} B/B$ has been calculated for the toroidal and the total poloidal field configurations used. The ratio

$$\frac{\nabla_{\perp} V}{V} \bigg/ \frac{\nabla_{\perp} B}{B}$$

amounted for these types of magnetic fields to about 0.1 in parts of the apparatus not too near to the cathode. From this estimate the drift velocity due to magnetic inhomogeneities will be about 10 times larger than the drift due to the electric field, so the electron motion seems to be controlled mainly by the magnetic field. However, not all components of the electric field are active in altering the form of the cut-off curves and, as a consequence, the importance of the electric field drift is smaller than given by this estimate. As seen from eqs. (1) and (2) an axisymmetric electric field will only change the cut-off condition to that for electrons with larger energy. An electric field symmetric with respect to the cathode might force the electrons into and out of a certain magnetic "surface" $rA_{\varphi} = \text{const.}$ in a way such that the net resulting motion is only a very slow drift out of the confinement region.

This result as well as the visual observation described in sec. 3 supports the argument that, almost all over the confinement volume, the effect due to the drift in crossed electric and magnetic fields is small compared to that produced by the magnetic field inhomogeneity.

VI. Conclusions

In general the present experiments have shown good agreement with the theory of motion of single charged particles. The absence of a complete cut-off in these investigations as well as in earlier research on the magnetron (Harvey 1943) has not yet been fully understood. As only very small currents are observed beyond the calculated cut-off value, the loss of particles is in any case not enhanced catastrophically. Instead a good confinement is likely to be secured if the magnetic field is made strong enough.

The experimental arrangement used in the investigation of the field from a current loop can be considered as a "three-dimensional magnetron". The cut-off is strongly pronounced and occurs to all electrodes in the apparatus, even to the suspensions. This seems to be due to the high mirror ratio by which most of the electrons will be reflected before reaching the suspensions. In a plasma experiment the same magnetic field configuration could possibly serve as a mirror machine with a high mirror ratio, and with a small loss cone.

Finally, the important conclusion can be drawn, that the toroidal field does not affect the cut-off curves. This agrees with the theoretical result given by eqs. (1) and (2) which predicts that the confinement should be controlled only by the poloidal part of the magnetic field.

ACKNOWLEDGEMENTS

The author is indebted to Dr. B. Lehnert for suggesting these investigations. The valuable assistance by Mr. J. Bergström and Mr. S. Holmberg is also gratefully acknowledged. This work has been made possible by a grant from the Swedish Atomic Council.

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REFERENCES

- GLASER, W., Grundlagen der Elektronenoptik. Wien 1952.
HARVEY, A. F., High frequency thermionic tubes. New York 1943.
LEHNERT, B., Nature 181, 331 (1958).
—— Journ. of Nuclear En. Part C 1, 40 (1959).

Tryckt den 3 november 1960

Uppsala 1960. Almqvist & Wiksells Boktryckeri AB

